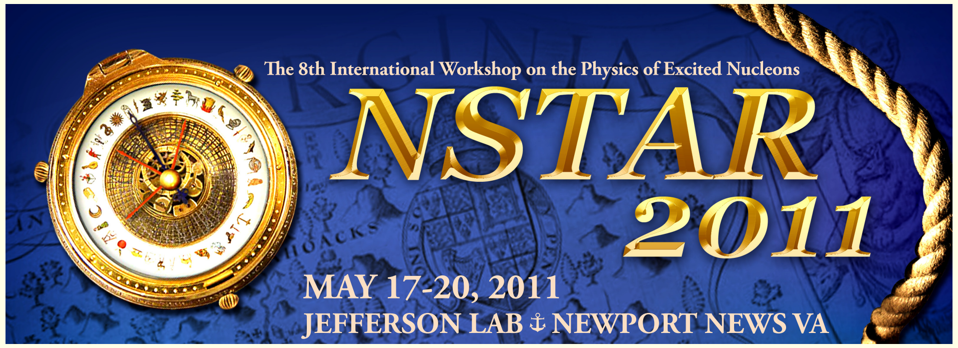

ORGANIZING THE SPIN-FLAVOR STRUCTURE OF BARYONS

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Outline

- Spin-flavor symmetry in baryons
- Breaking of SF symmetry: $1/N_c$, m_q ordering
- Ground state baryons
- Excited baryons: masses, photocouplings, partial widths
- Observations

Spin-flavor SU(6)

$$SU(3) \times SU(2) \subset SU(6)$$

SU(6): non-relativistic dynamical symmetry

Conserved currents only associated with $SU(3) \times SU(2)_{spin}$

Results from decoupling of spin in NRQM

Not good in mesons

Good in baryons

Phenomenological successes: Gursey-Radicati mass formula, F/D=2/3 (Skyrme: 5/9; phen: 0.58±0.04), magnetic moments, excited baryon multiplets and observables

SU(6) from QCD: emergent symmetry in large N_c

SU(6) from consistency in large N_c

for large N_c :

$$M_{baryon} = \mathcal{O}(N_c) \qquad g_{\pi BB'} = \mathcal{O}(\sqrt{N_c})$$

need contracted SU(6) symmetry in large N_c

Generators:

$$\{T^a,\ S^i,\ G^{ia}\}$$

Breaking of SU(6): expansion in $1/N_c$ and m_s-$m_{u,d}$

At large N_c SU(6) plays a key role in baryons:

is N_c =3 large enough?

Can only be tested through phenomenology and lattice QCD

$1/N_c$ at baryon level: expansion with effective operators built from generators of SU(6)

Bases of operators for masses, amplitudes, etc

$1/N_c$ order of effective n-body operators:

$$\nu = n - 1 - \kappa$$

Ground state baryon masses: 8 and 10

$$M_{GS} = c_1\, N_c + \frac{c_{HF}}{N_c}\,(S^2 - \frac{3}{4}N_c) - c_{\mathcal{S}}\frac{m_s - m_{u,d}}{\Lambda}\,\mathcal{S} + \mathcal{O}(1/N_c^2; m_s/N_c)$$

Gursey-Radicati

Effective coefficients encode the QCD dynamics

Parameter independent relations

Linear relations $$\frac{\partial}{\partial c_n}\sum_i a_i A_i = 0$$

Quadratic relations $$\frac{\partial^2}{\partial c_n \partial c_m}\sum_i a_i \Gamma_i = 0$$

Accurate to given order in expansions

Example: GS baryon masses

	$\Sigma - \Lambda = \mathcal{O}(m_s/N_c)$	74 MeV
GMO	$\Xi_8 - \Sigma_8 = \frac{1}{2}(3\Lambda - \Sigma_8) - N$	128 vs 141 MeV
ES	$\Sigma_{10} - \Delta = \Xi_{10} - \Sigma_{10}$	153 vs 145
”	$\Omega^- - \Xi_{10} = \Xi_{10} - \Sigma_{10}$	142 vs 145
8-10	$\Sigma_{10} - \Sigma_8 = \Xi_{10} - \Xi_8$	212 vs 195

Excited baryons

extend to SU(6)xO(3)

O(3) symmetry breaking is relatively small from phenomenology

Excited baryons organized in SU(6)xO(3) multiplets

$$[56, 0^+], [56, 2^+], [70, 1^-], etc$$

Configuration mixings organized in powers of $1/N_c$: expected to be small but poorly understood; new insights from lattice (HSC)

Sufficient phenomenology for useful $1/N_c$ analyses

[56,2$^+$] mass relations

$[56, 2^+]$	*masses*	$[MeV]$
State	$1/N_c$	*PDG*
$N_{3/2}$	1674 ± 15	1700 ± 50
$\Lambda_{3/2}$	1876 ± 39	1880 ± 30
$\Sigma_{3/2}$	1881 ± 25	(1840)
$\Xi_{3/2}$	2081 ± 57	
$N_{5/2}$	1689 ± 14	1683 ± 8
$\Lambda_{5/2}$	1816 ± 33	1820 ± 5
$\Sigma_{5/2}$	1920 ± 24	1918 ± 18
$\Xi_{5/2}$	1997 ± 49	
$\Delta_{1/2}$	1897 ± 32	1895 ± 25
$\Sigma_{1/2}$	2068 ± 52	
$\Xi_{1/2}$	2237 ± 88	
$\Omega_{1/2}$	2408 ± 127	
$\Delta_{3/2}$	1906 ± 27	1935 ± 35
$\Sigma'_{3/2}$	2061 ± 44	(2080)
$\Xi'_{3/2}$	2216 ± 76	
$\Omega_{3/2}$	2373 ± 110	
$\Delta_{5/2}$	1921 ± 21	1895 ± 25
$\Sigma'_{5/2}$	2051 ± 37	(2070)
$\Xi'_{5/2}$	2181 ± 64	
$\Omega_{5/2}$	2313 ± 94	
$\Delta_{7/2}$	1942 ± 27	1950 ± 10
$\Sigma_{7/2}$	2036 ± 44	2033 ± 8
$\Xi_{7/2}$	2131 ± 76	
$\Omega_{7/2}$	2229 ± 110	

Mass operators @ O(1/N$_c$, m$_q$):
1 LO, 2 NLO, 3 SU(3) breaking
22 PIRs; 7 can be tested

	$\mathcal{O}(\Lambda/N_c^2)$		$Exp[MeV]$
	$\frac{1}{2}(\Delta_{5/2} - \Delta_{3/2} - N_{5/2} + N_{3/2})$	$=$	-12 ± 33
	$\sqrt{\frac{2}{53}}(\Delta_{7/2} - \Delta_{5/2} - \frac{7}{5}(N_{5/2} - N_{3/2}))$	$=$	15 ± 15
	$\frac{1}{2\sqrt{5}}(\Delta_{7/2} - \Delta_{1/2} - 3(N_{5/2} - N_{3/2}))$	$=$	24 ± 34
	$\frac{1}{2\sqrt{3}}(\Lambda_{5/2} - \Lambda_{3/2} + \Sigma_{5/2} - \Sigma_{3/2} - 2(\Sigma'_{5/2} - \Sigma'_{3/2})$	$=$	11 ± 36
	$\frac{1}{\sqrt{218}}(7\,\Sigma'_{3/2} + 5\,\Sigma_{7/2} - 12\,\Sigma'_{5/2})$	$=$	-7 ± 38
	$\frac{1}{\sqrt{57}}(4\,\Sigma_{1/2} + \Sigma_{7/2} - 5\,\Sigma'_{3/2})$		
	$\mathcal{O}(m_s/N_c^2)$		$Exp[MeV]$
	$\frac{1}{\sqrt{3346}}(8\Lambda_{3/2} - 8N_{3/2} + 37\Lambda_{5/2} - 22N_{5/2} - 15\Sigma_{5/2} - 30\Sigma_{7/2} + 30\Delta_{7/2})$	$=$	8.5 ± 12
	$\frac{1}{2\sqrt{13}}(\Lambda_{5/2} - \Lambda_{3/2} + 3(\Sigma_{5/2} - \Sigma_{3/2}) - 4(N_{5/2} - N_{3/2}))$	$=$	34 ± 34
(GMO)	$2(N + \Xi)$	$=$	$3\,\Lambda + \Sigma$
(EQS)	$\Sigma - \Delta$	$=$	$\Xi - \Sigma = \Omega - \Xi$

$[70,1^-]$ mass relations

Mass operators: 1 LO, 12 NLO, 3 SU(3) break;
15 PIRs

Masses [MeV]		
State	*Exp*	*Large N_c*
$N_{1/2}$	1538 ± 18	1541
$\Lambda_{1/2}$	1670 ± 10	1667
$\Sigma_{1/2}$	(1620)	1637
$\Xi_{1/2}$	(1690)	1779
$N_{3/2}$	1523 ± 8	1532
$\Lambda_{3/2}$	1690 ± 5	1676
$\Sigma_{3/2}$	1675 ± 10	1667
$\Xi_{3/2}$	1823 ± 5	1815
$N'_{1/2}$	1660 ± 20	1660
$\Lambda'_{1/2}$	1785 ± 65	1806
$\Sigma'_{1/2}$	1765 ± 35	1755
$\Xi'_{1/2}$		1927
$N'_{3/2}$	1700 ± 50	1699
$\Lambda'_{3/2}$		1864
$\Sigma'_{3/2}$		1769
$\Xi'_{3/2}$		1980
$N_{5/2}$	1678 ± 8	1671
$\Lambda_{5/2}$	1820 ± 10	1836
$\Sigma_{5/2}$	1775 ± 5	1784
$\Xi_{5/2}$		1974
$\Delta_{1/2}$	1645 ± 30	1645
$\Sigma''_{1/2}$		1784
$\Xi''_{1/2}$		1922
$\Omega_{1/2}$		2061
$\Delta_{3/2}$	1720 ± 50	1720
$\Sigma''_{3/2}$		1847
$\Xi''_{3/2}$		1973
$\Omega_{3/2}$		2100
$\Lambda''_{1/2}$	1407 ± 4	1407
$\Lambda''_{3/2}$	1520 ± 1	1520

$\mathcal{O}(m_s/N_c^2; m_s^2)$
$\frac{1}{\sqrt{16930}}\left(14(\tilde{\Lambda}_{3/2} + \tilde{\Lambda}'_{3/2}) + 63\tilde{\Lambda}_{5/2} + 36(\tilde{\Sigma}_{1/2} + \tilde{\Sigma}'_{1/2}) - 68(\tilde{\Lambda}_{1/2} + \tilde{\Lambda}'_{1/2}) - 27\tilde{\Sigma}_{5/2}\right)$
$\frac{1}{\sqrt{1570}}\left(14(\tilde{\Sigma}_{3/2} + \tilde{\Sigma}'_{3/2}) + 21\tilde{\Lambda}_{5/2} - 9\tilde{\Sigma}_{5/2} - 18(\tilde{\Lambda}_{1/2} + \tilde{\Lambda}'_{1/2}) - 2(\tilde{\Sigma}_{1/2} + \tilde{\Sigma}'_{1/2})\right)$
$\frac{1}{\sqrt{8066}}\left(14\tilde{\Sigma}''_{1/2} + 49\tilde{\Lambda}_{5/2} + 23(\tilde{\Sigma}_{1/2} + \tilde{\Sigma}'_{1/2}) - 45(\tilde{\Lambda}_{1/2} + \tilde{\Lambda}'_{1/2}) - 19\tilde{\Sigma}_{5/2}\right)$
$\frac{1}{2\sqrt{695}}\left(14\tilde{\Sigma}''_{3/2} + 28\tilde{\Lambda}_{5/2} + 11(\tilde{\Sigma}_{1/2} + \tilde{\Sigma}'_{1/2}) - 27(\tilde{\Lambda}_{1/2} + \tilde{\Lambda}'_{1/2}) - 10\tilde{\Sigma}_{5/2}\right)$

GMO $8_{3/2}$

$$2(N_{3/2} + \Xi_{3/2}) - 3\Lambda_{3/2} - \Sigma_{3/2} = -19 \pm 26\ MeV$$

$\Xi_{1/2}(1690)$ BaBar 06

GMO gives $M_{\Xi_{1/2}} = 1779 \pm 30\ MeV$

[56,2$^+$] photocouplings $B_\lambda(M)$

All PIRs are accurate up to corrections 1/ N_c

Nucleons

$$\frac{\sqrt{2}\, p_{1/2}(1680) + \sqrt{2}\, n_{1/2}(1680)}{p_{3/2}(1680) + n_{3/2}(1680)} = 1 \qquad Exp: 0.19 \pm 0.16$$

E_2

$$-\frac{1}{\sqrt{3}} \frac{p_{1/2}(1720) + n_{1/2}(1720)}{p_{3/2}(1720) + n_{3/2}(1720)} = 1 \qquad Exp: -0.94 \pm 3.5$$

M_1, E_2

Deltas

$$\frac{\Delta_{1/2}(1950)}{\Delta_{3/2}(1950)} = \sqrt{3/5} = 0.77 \qquad Exp: 0.78 \pm 0.15$$

M_3, E_4

$$\frac{\sqrt{7/27}\, \Delta_{1/2}(1910) - \Delta_{1/2}(1920) + \Delta_{1/2}(1905)}{0.484\, \Delta_{1/2}(1950)} = 1 \qquad Exp = 1.84 \pm 0.60$$

$M_{1,3}$, $E_{2,4}$

$[70,1^-]$ photocouplings

PIRs for Nucleons

E_1

$$\frac{p_{1/2}(1535)+n_{1/2}(1535)}{p_{1/2}(1650)+n_{1/2}(1650)}\times\frac{\cos\theta_1+\sqrt{2}\sin\theta_1}{\sqrt{2}\cos\theta_1-\sin\theta_1}=-1 \qquad Exp=-1.82\pm 2.1$$

$$\theta_1=0.39$$

M_2 , E_3

$$\sqrt{2}\,\frac{p_{1/2}(1675)+n_{1/2}(1675)}{p_{3/2}(1675)+n_{3/2}(1675)}=1 \qquad Exp: 1.1\pm 0.3$$

Violations to Moorhouse rule: $^4N \not\to p\gamma$

- holds in single-quark transition model
- Moorhouse suppressed couplings

$$\underbrace{p_{1/2}(1650)}_{\theta_1} \qquad \underbrace{p_{1/2}(1700)}_{1-body,\ 2-body} \qquad \underbrace{p_{1/2}(1675)}_{2-body}$$

$[70,1^-]$ partial widths

$$\tilde{\Gamma}_i = \frac{\Gamma_i \, \Lambda^{2\ell_\pi}}{k_\pi^{1+2\ell_\pi}}$$

relations at LO

$$\frac{\partial^2}{\partial c_n \partial c_m} \sum_i a_i \tilde{\Gamma}_i = 0$$

LO: S-wave 4 operators, 1 1-body
D-wave 5 operators, "

relations valid to N_c^0 & no SU(3) breaking

NLO corrections to $\tilde{\Gamma}_i$ ~60%

S-wave relations

			$S-wave$		
$State$	J	$Mass$ (MeV)	Γ_{total} (MeV)	$Channel$	Γ (MeV)
$N(1535)$	$1/2$	1535	150(25)	$N\pi$	67.5(18.8)
				$N\eta$	78.75(17.3)
$N(1520)$	$3/2$	1520	112.5(12.5)	$\Delta\pi$	9.56(4.1)
$N(1650)$	$1/2$	1655	165(20)	$N\pi$	128(32.8)
				$N\eta$	10.7(9.2)
				ΛK	11.55(6.7)
$\Lambda(1670)$	$1/2$	1670	37.5(12.5)	$N\bar{K}$	9.4(3.6)
				$\Lambda\eta$	6.56(3.56)
				$\Sigma\pi$	15(7.5)
$\Lambda(1800)$	$1/2$	1800	300(100)	$N\bar{K}$	97.5(39.5)
$\Lambda(1405)$	$1/2$	1406	50(2)	$\Sigma\pi$	50(2)
$\Sigma(1750)$	$1/2$	1750	110(50)	$N\bar{K}$	27.5(20.7)
				$\Sigma\pi$	4.4(4.4)
				$\Sigma\eta$	38.5(28.1)
$\Delta(1620)$	$1/2$	1630	142.5(7.5)	$N\pi$	35.6(7.4)
$\Delta(1700)$	$3/2$	1700	300(100)	$\Delta\pi$	112.5(53)

$S-wave\ Relation$	$Exp\ \ Test$
$\frac{N(1650)\to\pi N}{N(1535)\to\eta N} = \frac{N(1535)\to\pi N}{N(1650)\to\eta N}$	$0.6 \pm 0.2\ vs\ 4.4 \pm 4.0$
$\frac{N(1650)\to\eta N}{\Sigma(1750)\to\eta\Sigma} = 1$	0.12 ± 0.14
$\frac{N(1535)\to\eta N}{\Lambda(1670)\to\eta\Lambda} = 1$	5.4 ± 3.2
$\frac{\Delta(1620)\to\pi N}{\Delta(1700)\to\pi\Delta} = 2/5$	0.29 ± 0.15
$\frac{N(1535)\to\pi N}{\Lambda(1670)\to\pi\Sigma} = 1$	4.4 ± 2.5
$\frac{N(1650)\to\pi N}{\Lambda(1670)\to\eta\Lambda} = \frac{\Lambda(1670)\to\pi\Sigma}{N(1650)\to\eta N}$	$2.1 \pm 0.9\ vs\ 0.8 \pm 0.6$
$\frac{N(1650)\to\pi N}{N(1535)\to\eta N} = \frac{N(1535)\to\pi N}{N(1650)\to\eta N}$	$0.6 \pm 0.2\ vs\ 4.4 \pm 4.0$
$\frac{N(1535)\to\pi N}{N(1650)\to\pi N}$	$\theta_1 = 0.30 \pm 0.08 \quad 1.6 \pm 0.08$
$\frac{N(1535)\to\pi N}{\Delta(1620)\to\pi N}$	$\theta_1 = 0.33 \pm 0.08 \quad 1.57 \pm 0.08$
$\frac{N(1535)\to\eta N}{N(1650)\to\eta N}$	$\theta_1 = 0.68 \pm 0.14 \quad 1.22 \pm 0.14$
$\frac{N(1520)\to\pi\Delta}{\Delta(1700)\to\pi\Delta}$	$\theta_3 = 2.48 \pm 0.08 \quad 2.96 \pm 0.09$

D-wave relations

$D - wave$

$State$	J	$Mass$ (MeV)	Γ_{total} (Mev)	$Channel$	Γ (MeV)
$N(1535)$	1/2	1535	150(25)	$\Delta\pi$	0.75(0.75)
$N(1520)$	3/2	1520	112.5(12.5)	$N\pi$	67.5(9.4)
				$\Delta\pi$	13.5(2.7)
$N(1650)$	1/2	1655	165(20)	$\Delta\pi$	6.6(5.0)
$N(1700)$	3/2	1700	100(50)	$N\pi$	10(7.1)
				ΛK	1.5(1.5)
$N(1675)$	5/2	1675	147.5(17.5)	$N\pi$	59(10.2)
				ΛK	0.74(0.74)
$\Lambda(1690)$	3/2	1690	60(10)	$N\bar{K}$	15(3.9)
				$\Sigma\pi$	18(6.7)
$\Lambda(1830)$	5/2	1830	85(25)	$N\bar{K}$	5.5(3.4)
				$\Sigma\pi$	46.75(21.9)
				$\Sigma_{10}\pi$	6.37(6.4)
$\Lambda(1520)$	3/2	1519	15.6(1)	$N\bar{K}$	7.02(0.5)
				$\Sigma\pi$	6.55(0.45)
$\Sigma(1670)$	3/2	1670	60(20)	$N\bar{K}$	6.0(2.7)
				$\Lambda\pi$	6.0(3.6)
				$\Sigma\pi$	27(12.7)
$\Sigma(1775)$	5/2	1775	120(15)	$N\bar{K}$	48.0(7.0)
				$\Lambda\pi$	20.4(4.4)
				$\Sigma\pi$	4.2(1.9)
				$\Sigma_{10}\pi$	12(2.8)
$\Delta(1620)$	1/2	1630	142.5(7.5)	$\Delta\pi$	64.1(21.6)
$\Delta(1700)$	3/2	1700	300(100)	$N\pi$	45(21.2)
				$\Delta\pi$	12.0(9.8)

$D - wave\ Relation$	$Exp\ \ Test$
$\frac{N(1675)\to\pi N}{\Lambda(1830)\to\pi\Sigma} = 1$	0.92 ± 0.46
$\frac{\Sigma(1670)\to\pi\Lambda}{\Sigma(1670)\to\pi\Sigma} = 1/2$	0.12 ± 0.10
$\frac{\Sigma(1775)\to\pi\Lambda}{\Sigma(1775)\to\pi\Sigma} = 1/2$	3.1 ± 1.6
$\frac{\Sigma(1775)\to\pi\Sigma}{\Sigma(1775)\to\pi\Sigma_{10}} = 8/7$	1.3 ± 0.6
$\frac{2\Delta(1620)\to\pi\Delta+\Delta(1700)\to\pi\Delta}{8\Delta(1700)\to\pi N+N(1675)\to\pi N} = 1$	2.9 ± 1.2
$\frac{\frac{2}{9}\ N(1535)\to\pi\Delta+\frac{2}{9}\ N(1650)\to\pi\Delta+\frac{20}{3}\ \Delta(1620)\to\pi\Delta}{16\Delta(1700)\to\pi N+15N(1675)\to\pi N} = 1$	2.6 ± 1.2
$\frac{\frac{1}{36}\ N(1520)\to\pi N+\frac{1}{36}\ N(1700)\to\pi N+\frac{5}{12}\ \Delta(1620)\to\pi\Delta}{\Delta(1700)\to\pi N+N(1675)\to\pi N} = 1$	2.5 ± 1.2

OBSERVATIONS

- $1/N_c$ expansion justifies use of SU(6) symmetry in baryons

- Analyses of various baryon observables to NLO gives consistent picture in most cases: natural size NLO corrections

- PIRs provide a useful test at given order; some gold plated predictions to 1/Nc; model independent predictions to given order

- New insights on spin-flavor structure of baryons emerging from Lattice QCD (HSC)! - promising for testing the 1/Nc expansion

- Open issues: understanding physics encoded in effective constants; configuration mixings; beyond the Algebra - include EFT dynamics (being done in ChPT)


Thank you